MAXIMUM LIKELIHOOD METHODS FOR TIME-RESOLVED IMAGING THROUGH TURBID MEDIA

Brian Eriksson and Robert Nowak

University of Wisconsin - Madison
Department of Electrical and Computer Engineering
1415 Engineering Drive, Madison, WI 53706
bceriksson@wisc.edu, nowak@engr.wisc.edu

ABSTRACT

Recently technological advances now enable time-gated acquisitions of photons at very fast rates. This can allow one to separate scattered and unscattered photons in transillumination imaging. Time-resolved transillumination (TRT) imaging opens the door to an new type of imaging through turbid (scattering) media such as soft tissue and fog/smoke, and exciting potential applications in bioimaging and surveillance. This paper proposes a novel Maximum Likelihood based approach to TRT image reconstruction.

***Index Terms*—** Image reconstruction, Poisson processes, EM Algorithm, Time-resolved Transillumination Imaging

1. TIME-RESOLVED TRANSILLUMINATION IMAGING

Recently technological advances now enable time-gated acquisitions of photons at very fast rates, fast enough to separately collect unscattered (first arrival) and scattered (later arrival) photons in transillumination imaging [1]. We refer to this technology as time-resolved transillumination (TRT) imaging, which is described in more detail in the next paragraph and depicted in Fig. 1. TRT opens the door to an new type of imaging turbid (scattering) media (e.g., soft tissue, fog/smoke). The ability to separately detect unscattered or *ballistic* photons can enable much higher resolution imaging than possible using conventional imaging devices, and this has exciting potential applications in bioimaging and surveillance. However, the number of ballistic photons decays exponentially fast as the thickness/depth of the turbid medium increases. Therefore, the high resolution information that is available is also in a very low SNR regime. This paper explores a novel approach to TRT image reconstruction.

In more detail, the TRT imaging problem involves of photons traveling through a turbid medium from an source, through an object plane, and then onto an imaging plane as depicted in Fig. 1. Photons traveling through a scattering medium can be roughly classified into three types: ballistic, snake, and diffuse. Ballistic photons experience no scattering and travel in a direct line of sight arriving first at detectors in the image plane. Because of the lack of scattering, ballistic photons retain their spatial information and arrive at the imaging plane at the same relative location as sent from the object plane. Snake photons experience some slight scattering through the medium, this scattering will cause these photons to arrive later than the ballistic photons and likely in a slightly different location than sent from the object plane. Diffuse photons experience large amounts of scattering and arrive at the image plane having lost most of their point of origin information. Due to the large number of scattering events through the medium, the diffuse photons will travel the farthest distance to the image plane, and therefore will arrive after the snake and ballistic photons. While the inherent spatial information decreases in order of ballistic, snake and diffuse photons, the number of photons (and hence inherent SNR) increases in the same order. So we are face with high resolution, low SNR data at one extreme (ballstic), and low resolution, high SNR data at the other (diffuse). Furthermore, the diffusion and SNR parameters, which characterize the underlying point spread function (PSF), are not known precisely in practice.

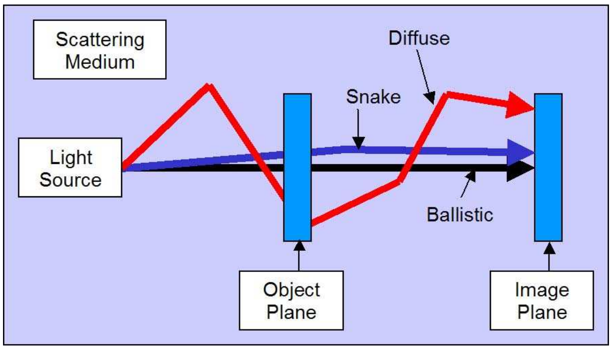

Fig. 1. Example of Photons Through a Scattering Medium

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The TRT image reconstruction problem is essentially a statistical inverse problem (a particular form of photon-limited image reconstruction), but to the best of our knowledge our work here is the first to formally pose it as such. Due to uncertainties in the PSF, the TRT problem somewhat related to blind image deconvolution (BID). While BID is a well-studied problem, there are several unique aspects in the TRT imaging problem that make it quite different from standard BID problems. In particular, the distinctive features of TRT imaging include the photon-limited nature of the data, the time-gated data acquisition which in effect yields information at multiple spatial resolutions that can be fairly well characterized via a diffusion equation, and most importantly the availability of "unblurred" data corresponding to the ballistic photons. For these reasons "off-the-shelf" BID algorithms are not directly applicable to TRT image reconstruction.

This paper is organized as follows. In Section 2 we propose a statistical model for TRT imaging through a homogeneous turbid medium. In Section 3 we review a multiscale Poisson denoising technique that can be applied directly to the ballistic data, and will also be an integral component of our overall reconstruction procedure. In Section 4 we show that an Expectation-Maximization (EM) algorithm based on the Poisson denoising scheme can be used to solve the image reconstruction problem when one has perfect knowledge of the scattering properties of the medium. In Section 5 we propose a novel scheme (based on the EM algorithm) for computing the joint Maximum Penalized Likelihood Estimate of the underlying image intensity and key diffusion and SNR parameters of the scattering environment. Section 6 evaluates the performance of our scheme in simulations and concluding remarks are made in Section 7.

2. A STATISTICAL MODEL OF TRANSILLUMINATION IMAGING

The basic statistical model we propose for TRT imaging through homogeneous turbid media is as follows. Assume that we have k time-resovled "snapshots" , each acquired over disjoint time intervals $T_1, \ldots, T_k$, with T_1 denoting the ballistic time interval. Assume that these intervals form a uniform partition of the overall observation interval T. Let $X_1, \ldots, X_k$ denote the photon data acquired in each interval, respectively. Specifically, the data X_i are acquired in the form of an n-pixel image, and for our mathematical exposition we assume that the columns of this image are "stacked" to form an $n \times 1$ vector. Each pixel value in X_i is simply the number of photons detected at the corresponding location during the time interval T_i. Each image is Poisson distributed according to the following model:

$$X_i \sim \text{Poisson}(\alpha_i P_i \lambda), \quad i = 1, \ldots, k, \qquad (1)$$

where λ denotes the underlying $n \times 1$ image intensity function, P_i denotes the $n \times n$ photon transition matrix from the emission (source) plane to the detection (image) plane, and $\alpha_i > 0$ is a scalar gain factor. The transition matrices are functions of time and a scalar diffusion bandwidth parameter denoted by σ^2. In particular, according to the basic physics of photon propagation through turbid media [1], row s of P_i is a probability mass function modeled by a sampled Gaussian density with mean s and variance proportional to $\sigma^2 t_i$, where t_i denotes the midpoint of the i-th acquisition time-interval. Thus, the transition matrices are parametric functions of the form $P_i = P(\sigma^2 t_i)$. We assume that λ is normalized such that the intensities in λ sum to one (i.e., $\sum_{j=1}^{n} \lambda_j = 1$). Also the transition matrices are normalized so that $\mu_i = P_i \lambda$ also satisfies $\sum_{j=1}^{n} \mu_{i,j} = 1$. With these normalizations, it is easy to verify that the "total" intensity (integrated spatially over the image plane and temporally over the i-th time-interval) is α_i. Furthermore, we adopt the convention that the ballistic image resolution is the finest spatial resolution available and assume that $P_1 = I_{n \times n}$, the $n \times n$ identity matrix. We also assume that the images acquired in each time-interval are statistically independent, and so the joint distribution function of the entire data record $X = [X_1^T, \ldots, X_k^T]^T$ (the superscript T denotes matrix transposition) is

$$X \sim \text{Poisson}(P\lambda), \qquad (2)$$

where P is the $kn \times n$ transition matrix obtained by stacking the matrices $\alpha_1 P_1, \ldots, \alpha_k P_k$; i.e., $P = [\alpha_1 P_1^T, \ldots, \alpha_k P_k^T]^T$.

Let us constrast this imaging system with a conventional system in which the photon detections are not time-resolved. In this case, we acquire the aggregated photon image $X_a = X_1 + \cdots + X_k$ which obeys the model

$$X_a \sim \text{Poisson}(P_a \lambda), \qquad (3)$$

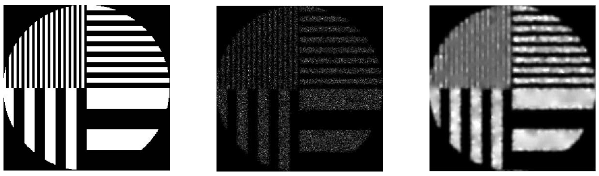
where $P_a = \sum_{i=1}^{k} \alpha_i P_i$. We will see that the extra "information" available in the time-resolved photon acquisition can significantly improve our ability to estimate the underlying intensity λ.

3. PHOTON-LIMITED IMAGE DENOISING

The ballistic photon image X_1 has high spatial resolution but extremely poor SNR due to the very limited number of ballistic photons. As a starting point for our work, let us consider the problem of estimating the underlying image intensity based on the ballistic photon data alone. This boils down to a Poisson image "denoising" problem, which has recently received a significant amount of attention in the image processing and statistics literature.

One state-of-the-art Poisson denoising scheme is based on the recursive dyadic partition (RDP) framework proposed in [2]. This scheme is a Poisson analog of the more familiar wavelet denoising methods developed for the classical "signal+noise" model. Also like wavelet denoising, additional improvements in denoising quality are possible using a translation-invariant version of the basic approach [3, 4], which can be computed in $O(n \log n)$ operations.

The Poisson denoising method will be an integral component of our EM algorithm for time-resolved transillumination imaging. The EM algorithm optimally combines information from the entire record time-resolved photons (i.e., from the ballistic, quasidiffuse and diffuse limits). But before moving on, let us illustrate the performance of the method by estimating the underlying image intensity using only the ballistic photon data. Figure 2 depicts the results of denoising a ballistic photon image using the methodology described above (specifically, we employ the translation-invariant Haar estimation scheme described in [4] and implemented in the superb Matlab package developed by Prof. Rebecca Willett [5]). Note that the denoising method results in an intensity estimate that reduces "noise" while preserving edges and other details.



(a) Image Intensity (b) Ballistic Data (c) MPLE

Fig. 2. Example of ballistic photon image MPLE denoising.

4. AN EM ALGORITHM FOR IMAGE RECONSTRUCTION

If the attenuation factors $\{\alpha_i\}$ and scattering matrices $\{P_i\}$ are known, then the MLE of λ given X (or given X_a) can be computed using the well-known Expectation-Maximization (EM) algorithm [6]. For the general problem, the standard E and M steps are computed using X and P, while in the case of aggregated photon data the steps employ X_a and P_a. The algorithm is initialized with a starting guess for MLE of λ (e.g., all pixels set to 1).

Under mild conditions, the sequence of estimates converges to an MLE solution. Unfortunately, because P is usually poorly-conditioned, the exact MLE solution is usually undesirable. For example, in the case of the ballistic data alone, the MLE of λ is simply X_1, which as seen from Figure 2, is highly variable and typically has a very poor MSE. So, instead of seeking the MLE we aim to recover a Maximum Penalized Likelihood Estimate (MPLE), using the Poisson denoising criterion described in Section 3 as our penalty term. This MPLE approach was first proposed in [7]. The MPLE can also be computed using the EM algorithm. In this case, the E-Step remains the same as above and the M-Step is computed by applying the translation-invariant denoising algorithm [4, 5] to the usual M-step result prior to re-computing the E-Step (see [7] for further details).

5. ADAPTING TO UNKNOWN TURBID MEDIA

One of the major challenges in practical imaging problems is that the characteristics of the turbid medium are usually not known precisely. In particular, the values of system parameters $\{\alpha_i\}$ and σ^2 are unknown and therefore must be estimated along with λ. We can formulate this as a joint MLE problem, seeking to find values of the system parameters and λ which jointly maximize the Poisson likelihood function (or penalized likelihood). At first glance, it may appear that this joint MLE problem may be intractable, but it turns out to have a rather simple solution which is one of the main contributions of this paper.

The basic solution approach is as follows. First, the MLEs of the gain factors $\{\alpha_i\}_{i=1}^k$ can be computed separately from λ and σ^2 due to the following observation. Consider the statistics

$$S_i = \sum_{j=1}^{n} X_{i,j}, \;\; i = 1, \ldots, k,$$

the subscript j indexes the pixels in each image. These statistics are simply the total photon counts for each image. Due to the normalization of λ and the matrices $\{P_i\}$ in our model, it follows that

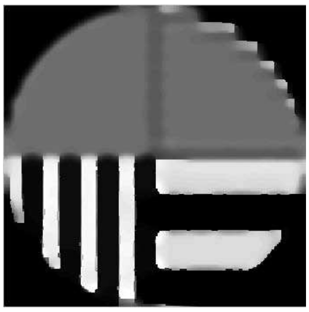
$$S_i \;\sim\; \text{Poisson}(\alpha_i), \;\; i = 1, \ldots, k.$$

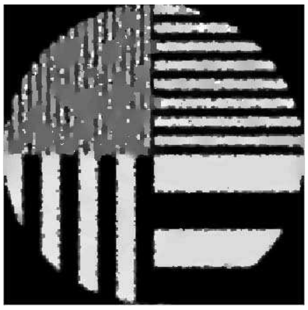
It is well-known that the conditional distribution of X_i given S_i is multinomial with parameters $\mu_i = P_i\lambda$ (see [2]). Therefore, the likelihood factorizes into Poisson factors, each involving one pair (α_i, S_i), and multinomial factors, each involving λ and one triple (P_i, X_i, S_i). Consequently, the MLEs of the gain factors $\{\alpha_i\}$ can be obtained separately from λ and σ^2 and are given by the simple formula $\widehat{\alpha_i} = S_i$, $i = 1, \ldots, k$. Now recall that the matrices $\{P_i\}$ are parametric in σ^2. To find the MLEs of λ and σ^2 we consider a range of candidate values for σ^2 and for each one we use the EM algorithm described above (with each α_i set to its MLE $\widehat{\alpha_i}$) to compute the MPLE, denoted by $\widehat{\lambda}(\sigma^2)$, and the corresponding maximum penalized likelihood value, denoted $L(\sigma^2)$. This can be done exhaustively (over a discretized range of σ^2 values) or systematically (assuming unimodality of the maximum penalized likelihood as a function of σ^2 and employing a bisection method). The value of σ^2 that results in the highest penalized likelihood value $L(\sigma^2)$ is the MLE of σ^2, denoted by $\widehat{\sigma^2}$. The MLE of λ is then $\widehat{\lambda}(\widehat{\sigma^2})$.

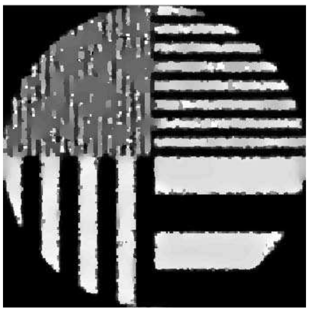
6. A TRT IMAGING EXAMPLE

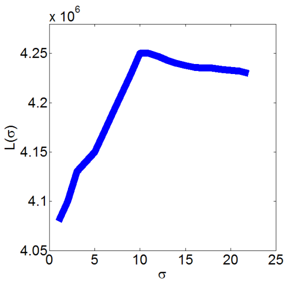
The potential of the proposed EM algorithm is evaluated in the following simulation study. Using the λ intensity function depicted in Figure 2a, a ballistic image is generated using low photon count Poisson data and a diffuse image is generated by first blurring λ using a high variance blurring kernel and then generating a large number of photons from the blurred intensity function. The reconstruction based only on the ballistic photon data is depicted in Figure 2c, and the reconstruction using only the diffuse image and assuming perfect knowledge of the blurring variance σ^2 is depicted in Figure 3a). Figure 3b shows the reconstruction based on both ballistic and diffuse photon data and assuming perfect knowledge of attenuation factors (α_1, α_2) and blurring variance (σ), using the EM algorithm described in Section 4. Using the adaptive MLE scheme described in Section 5, the attuenation factors and blurring variance were jointly estimated along with λ, and the resulting intensity estimate shown in (Figure 3c). Table 1 summarizes the reconstution errors over 10 independent trials of the experiment. As expected, the combined (ballistic+diffuse) oracle reconstruction performs best, with the combined reconstruction with ML estimation of the gain and diffusion parameters close behind.

The algorithm developed in Section 5 the final reconstruction is the estimate with largest likelihood value chosen from a set of reconstructed images generated using different σ^2 values. In all our experiments, the likelihood appears to be unimodal in σ^2 (one such likelihood is seen in (Figure 3d)), allowing for systematic searches such as a bisection method. We conjecture that the likelihood is always unimodal in σ^2, but we have not yet proved this result.


(a) MPLE Diffuse


(b) MPLE Ballistic + Diffuse


(c) MPLE Ballistic + Diffuse w/ Unknown parameters



(d) Log-Likelihood Plot

Fig. 3. Examples of TRT image reconstruction.

Table 1. Summary of TRT image reconstruction errors.

Image	*PSNR (dB)*
Ballistic Image	4.08
Diffuse Image	7.00
MPLE Ballistic	11.90
MPLE Diffuse	9.42
MPLE Ballistic + Diffuse	12.92
MPLE Ballistic + Diffuse w/ unknown params	12.71

7. CONCLUSIONS

This paper proposed a novel MLE reconstruction algorithm for TMT imaging. The algorithm is based on a combination of the EM algorithm and Poisson denoising. Our simulation study demonstrates the potential of our approach, in particular indicating the added benefit of optimally fusing ballistic and diffuse photon data. Our future work includes the application of this theory to real-data experiments, detailed analysis of fundamental performance limits in TRT imaging, and extensions to inhomogeneous media.

8. REFERENCES

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